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THE MEDIAN AND DISTANCE MEASURES OF SELF-CENTRED GRAPHS

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ABSTRACT

The median of a graph is the set of vertices which has the minimum distance. The set defines the Median sub graph. In this paper, firstly we search the median of a self-centred graph. After we define the new graph operation that is called thick product. We prove some theorems on the median of the products of two self-centred graphs. Specially, we give the some results on the median of hypercube, because of hypercube is an important computer network topology and it is a self-centred graph.
Keywords: Graph Theory, Self-centred Graphs, Distance Measures in Graphs, Communication Network Topology, Hypercubes.

KENDİNİ MERKEZLEYEN ÇİZGELERİN UZAKLIK ÖLÇÜMLERİ VE ORTASI

ÖZET

Bir çizgenin ortası,minimum uzaklıklı düğümlerinin kümesidir.Bu düğümlerin kümesi çizgenin orta çizgesini tanımlar.Bu çalışmada önce kendini merkezleyen çizgelerin ortası çalışılmıştır.Yeni bir çizge işlemi olarak yoğun çarpım tanımlanmıştır.Sonra, iki kendini merkezleyen çizgenin çarpım çizgelerinin ortası üzerine teoremler ispatlanmıştır.Özel olarak da kendini merkezleyen çizgeler sınıfından ve önemli bir ağ topologisi olan hiperküb çizgelerin ortası üzerine sonuçlar verilmiştir.
Anahtar Sözcükler: Çizge Kuramı, Kendini-merkezleyen Çizgeler, Çizgelerde Uzaklık Ölçümleri, İletişim Ağları Topolojisi, Hiperkübler.

1.INTRODUCTION

A graph G is denoted by G = (VG), E (G)), where V (G) and E (G) are vertices and edges sets of G, respectively. p denotes the number of vertices and q denotes the number of edges of the graph G.

The length of a shortest v-u path in a connected G is called *the distance* from a vertex v to a vertex u in a connected graph G.d(v,u) denotes the distance between v and u. *The eccentricity* e(v) of a vertex v in a connected graph G is the distance between v and a vertex farthest from v in G.The minimum and maximum eccentricities among the vertices of G are called the radius *radG* and the diameter *diamG* of G.The vertex v is a central vertex of G if e(v) = radG and the centre C(G) is the set of all central vertices.Thus,the centre consists of all vertices having minimum eccentricity. The central subgraph of a graph G is the subgraph induced by centre C (G). The vertex v is a *peripheral vertex* if e (v) =*diam*G and the periphery is the set of all such vertices. For a vertex v, each vertex at distance e (v) from v is an eccentric vertex for v.

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The status of a vertex v is defined as d (v) = $\sum\left\{d(v,u)\middle|v\in V(G)\right\}$ .We will call it the distance of v. Because d (v) is the sum of distance from v to all other vertices (v) is also called the total distance .For a graph G of order $\left|V(G)\right|=p$,*the average distance* from v to another vertex is $\frac{d(v)}{p}$. The *median* of G is $\left\{u\in V(G)\middle|d(u)\leq d(w)\,for\ all\ w\in V(G)\right\}$,the set of vertices of minimum (average) distance.M (G) denote the median subgraph, the subgraph induced by the set of median vertices [1, 2]. In paper [2] C.Smart- P.J.Slater work on centre , median and centroid sub graphs.

Graphs G_1 and G_2 have disjoint vertex sets V_1 and $V_2$and edges setsE_1 and E_2 respectively.

The Cartesian product $G = G_1 \times G_2$ has $V= V_1 \times V_2$,and two vertices (u_1 ,u_2) and (v_1,v_2) of G are adjacent if and only if either $u_1= v_1$ and $u_2v_2 \in E_2$ or $u_2= v_2$ and $u_1v_1 \in E_1$ [3,4].

Now, we define the thick product:
The thick product $G=G_1 \odot G_2$ has $V= V_1 \times V_2$,and two vertices (u_1 ,u_2) and (v_1,v_2) of G are adjacent if and only if either

i) $u_1= v_1$ and $u_2v_2 \in E_2$ or $u_2= v_2$ and $u_1v_1 \in E_1$

and

ii) $u_1v_1 \in E_1$ and $u_2v_2 \in E_2$.

In this section we give the results on accessibility number of some product graphs.

2. THE MEDIAN AND DISTANCE MEASURES OF SELF-CENTRED GRAPHS

In this section we give the definition *of self-centred graph* and the some theorems on distance measures of self-centred graphs.

Definition 2.1: A graph is *self-centred graph* if every vertex is in the centre.

Let G be self-centred graphs. Then it has the following properties:

Theorem 2.1[3]: If G is a self-centred graph with p vertices and q edges and its radius is 2, then $q \geq 2p-5$.

Theorem 2.2[3]: If G is a self-centred graph with p vertices, with Δ maximum degree and with radius r, then $\Delta \leq p-2r+2$.

Theorem 2.3: If G_1 and G_2 be two self-centred graph with *rad* r_1 and *rad* r_2.The Cartesian product graph $G_1 \times G_2$ is self-centred with *rad* $(r_1 + r_2)$.

Proof: Let G_1 and G_2 be two self-centred graph with *rad* r_1 and *rad* r_2.In the definition Cartesian product $G = G_1 \times G_2$,let be $|V_1|$=m and $|V_2|$=n. The graph $G_1 \times G_2$ has m number of graph G_2 and n number of graph G_1 . Any vertex (u_1 ,u_2) of $G_1 \times G_2$, u_1 connected to all vertices of G_2 with at most number of r_2 edges and u_2 connected to all vertices of G_1 at most number of r_1 edges. The distance of each vertex (u_1 ,u_2) to all the other vertex of graph $G_1 \times G_2$ is $r_1 + r_2$. Consequently, from the G_1 and G_2 are self-centred, The Cartesian product graph $G_1 \times G_2$ is self-centred with *rad* $(r_1 + r_2)$.

Result 2.1: $C(G_1 \times G_2) = V(G_1 \times G_2)$

Theorem 2.4: If G_1 and G_2 be two self-centred graphs with *rad* r_1 and *rad* r_2.Then, thick product graph $G_1 \odot G_2$ is self-centred with *rad* $(r_1 + r_2)$.

Proof: The proof of theorem is done the theorem 2.3.

Result 2.2: $C(G_1 \odot G_2) = V(G_1 \odot G_2)$.

Theorem 2.5: Let G_1 and G_2 be two self-centred graphs with *rad* r_1 and *rad* r_2.The *diameter* of the graphs $G_1 \times G_2$ and $G_1 \odot G_2$ are $(r_1 + r_2)$.

Proof: The proof of theorem is seen from Theorem 2.3.and Theorem 2.4.
Theorem 2.6: *The status* of each vertex of graph $G_1 x\ G_2$ or $G_1 \odot G_2$ is same.
Theorem 2.7: Let G_1 and G_2 be two self-centred graphs with *rad* r_1 and *rad* r_2.The median of the graphs $G_1 x\ G_2$ and $G_1 \odot G_2$ are themselves.
Thus, M $(G_1 x\ G_2)$ =V $(G_1 x\ G_2)$ and M $(G_1 \odot G_2)$ =V $(G_1 \odot G_2)$.

3. THE MEDIAN AND DISTANCE MEASURES OF HYPERCUBE

A hypercube network can be modelled by a hypercube graph whose vertices represent the processors and whose edges represent the links of hypercube network. A hypercube graph is a multidimensional mesh of vertices with exactly two vertices in each dimension. An n-dimensional hypercube consists $p=2^n$ vertices. A hypercube graph can be recursively constructed as follows: a single vertex is a 0-dimensional hypercube graph .1-dimensional hypercube graph is constructed by connecting two 0-dimensional hypercube graphs. In generally a (n+1)-dimensional hypercube is constructed by connecting the corresponding vertices of two n-dimensional hypercube. In this paper, n-dimensional hypercube network consider as a hypercube graph and it is denoted by the graph Q_n .Also, various graphs ca be embedded into the hypercube graphs[5,6,7,8,9,10].
In another hand, in graph theory the hypercube graph Q_n is defined as the Cartesian product of K_2 x Q_{n-1} ,where K_2 is a complete graph with two vertices.

A useful way of representing of a Hypercube graph is to replace each combination of binary variable labels by its decimal numbers equivalent. In figure 1, the vertices of hypercube graph Q_5 labels with the decimal numbers.

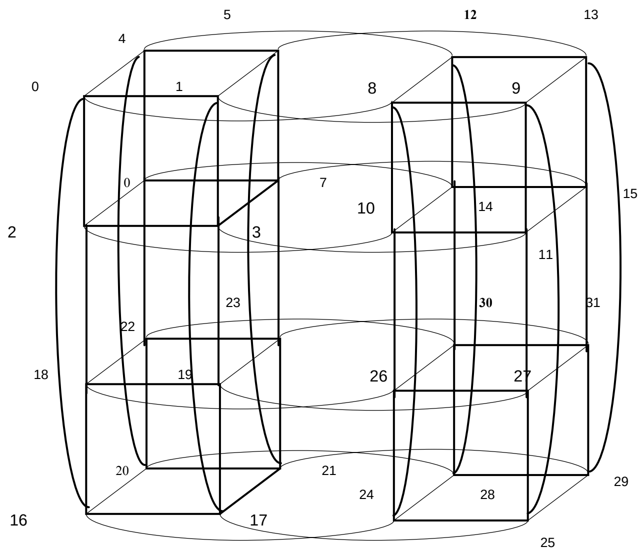

Figure 1. The hypercube graph Q_5

Every vertex of n-dimensional hypercube Q_n is marketed by n bit unique binary code. This cube has the coordinate's $k_1,k_2,\ldots,k_n$. The value of coordinate k_i is 0 or 1, for all i. In such coding of the vertex the neighbour vertices of n-dimensional cube are marked by d bit neighbour codes. Two d bit codes are called the neighbours if and only if they differ by the value of one bit. For example the codes 0110 and 0100 are neighbours. For the exact definition of a certain edge of cube it is sufficient to set the exact values of two respective coordinates For the vertices 000 and

001 ,cube has 00* edge, where * is called ''don't care symbol''. In such notation, the value of the coordinate k_i stands on i th position of Boolean expression that provides its unambiguous interpretation. The algebraic expression of whatever figure will call the vector of values of the coordinates.

Then, $k_i \in \{0,1,*\}$ for all $i = 1,2,...,n$. Including the values of m don't care coordinate, expresses the figure that is called m-cube. A vertex is 0-cube, an edge is a 1-cube, and a quadrangle is 2-cube.Some of the important properties of a Qn hypercube are as follows:

i) Two vertices are connected by an edge if and only if the binary representation of their labels differs at exactly one bit position.

ii) In a Q_n each vertex is connected to d other vertices.

iii) A Q_n hypercube graph can be partitioned into two Q_{n-1} sub cubes as follows: Select a bit position and group together all the vertices whose labels have a zero at the selected position; all of these vertices make up one partition, and the remaining vertices comprise the second partition. Since vertices labels have d bits, d such partitions exist.

iv) The vertex labels in a Q_n contain n bits. Fixing any k of these bits, the vertices that differ at the remaining n-k bit positions form a (n-k) –dimensional sub cube composed of 2^{n-k} vertices. Since k bits can be fixed in 2^k different ways, there are 2^k such sub cubes.

v) Consider the labels s and t of two vertices of in a hypercube graph Q_n. The total number of bit positions at which these two labels differ is called the Hamming Distance between them.

Theorem 3.1: n-dimensional hypercube graph Q_n is a self-centred graph.

Proof: The hypercube graph Q_n is defined as the Cartesian Product of K_2 x Q_{n-1} ,where K_2 is a complete graph with two vertices or a path two vertices and Q_1 is equal to K_2 .For $n \geq 2$, the graph Q_n recursively is obtained. The graph K_2 is self-centred graphs with *rad(*K_2) =1 .According to theorem 2.4 Q_n is a self-centred graph.

The following results and theorems can be seen from the theorems in section 2

Result 3.1: The radius of the graphs Q_n is *rad* (Q_n) =n.

Result 3.2: The *diameter* of the graphs Q_n is n.

Theorem 3.2: Each vertex of Qn is its centre vertex and C (Q_n) =V (Q_n).

Proof: The proof is seen from the theorem 3.1 and result 3.1.

Result 3.3: *The status* of each vertex of graph Qn is the same.

Theorem 3.3: The each vertex of Q_n is its median vertex and M (Q_n) =V (Q_n).Thus, Q_n is a median subgraph itself.

4. CONCLUSION

In this paper generally, we consider the self-centred graph. We define the thick product of two graphs. We work the median of graph products two self-centred graphs. We give the some theorems on median of the products graph .Also, the hypercube has received considerable attention and many hypercube-based machines have been available. In a large hypercube system, if we want to select the set of vertices that has the vertices of minimum (average) distance, what can we do? In this paper we show that any vertices set of hypercube has the same distance to all the other vertices.Now, we work to find using a graph distance algorithm the set of the vertices of minimum (average) distance in a faulty hypercube.

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