NATURAL AND FORCED VIBRATION OF THE THICK PLATE FABRICATED FROM THE SPATIALLY CURVED COMPOSITE

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ÜÇ BOYUTLU EĞRİSEL YAPIYA SAHİP KOMPOZİT MALZEMEDEN HAZIRLANMIŞ DİKDÖRTGEN KALIN PLAĞIN DOĞAL VE ZORLANMIŞ TİTREŞİMLERİ

ÖZET

Periyodik eğriliğe sahip kompozit malzemeden hazırlanmış kalın dikdörtgen plağın zorlanmış titreşimleri anizotropik bir yapı için elastisite teorisinin üç boyutlu kesin hareket denklemleri çerçevesinde araştırılmıştır. Bu plağın üst yüzeyinde düzgün yayılı normal kuvvetlerin etki ettiği ve bu kuvvetlerin zamana göre periyodik olarak değiştiği kabul edilmektedir. Plak malzeme yapısındaki periyodik eğriliğin plağın doğal frekansına ve gerilme dağılımına etkisi çalışılmaktadır. Sayısal sonuçlar üç boyutlu sonlu eleman modeli uygulanarak elde edilmiştir.

ABSTRACT

In the framework of the three-dimensional exact equations of motion of the theory of elasticity for an anisotropic body a forced vibration of the thick rectangular plate from the composite material with spatially periodically curved structure is investigated. It is assumed that on the upper face plane of the plate the uniformly distributed normal forces act and these forces periodically change with respect to time. The influence of the spatial periodical curving in the plate material structure to the fundamental frequency of the plate and on the stress distribution in that are studied. Numerical results are obtained by employing three-dimensional finite element modelling.

1. INTRODUCTION

In the mechanics of composite materials, problems associated with their structural features occupy a central position. One of the basic structural features of composite materials is curvature of the reinforcing elements. These curvatures may be due to the design features, as shown in Ref [1,2], or to technological processes resulting from the action of various factors, as shown in Ref [3,4]. For example, the structure of the woven-textile composites can be modelled through the periodical curved layers. However, to the local curving of the layers we can consider as a structural damage arising as a result of the technological procedures.

The successful practical use of artificially created composite materials is associated, to a considerable extent, with the determination of the stress-strain state in these materials, taking account of their basic features, particularly the distortion curving of the reinforcing elements. This distortion significantly influences the strength and strain properties of these materials.

There are two basic approaches to the study of the mechanics of composite materials with curved structure. First of them is continuum approaches which may be used to calculate the components of the stress-strain state for areas considerably greater in size than the curving; the influence of curving in the structure is taken into account by means of quantitative variation in the normalized mechanical characteristics. The review of the related investigations is given in Ref [5].

The second type of approach is the local one; developed later than the first, enabling the influence of reinforcing-element curving to be taken into account in calculating the components of the stress-strain state in areas comparable with, or smaller than the curving. These approaches were developed both in the framework of continuum theories and in the framework of a piecewise homogeneous body model. The review of the investigations made in the framework of the piecewise homogeneous body model is detailed in Ref [6,7].

As our present investigation has been made in the framework of the second type continuum approach, therefore here we consider brief review of corresponding investigations. We start with [8] in which the continuum theory was presented for unidirected fibrous or layered composites with spatially periodically or locally curved structures. The development of the theory [8] was also made in [9]. According to the theory [8,9], the mechanical relations of the composites with curved structures are modelled as the corresponding relations for the continuous inhomogeneous materials with normalized mechanical properties. The inhomogenity arises as a result of the curving and the function described that enters the elasticity relations.

In [10] in the framework of the theory [8,9] and with the use of the exact equations of the theory of elasticity the stress distribution in the plate-strip fabricated from the composite with periodically curved structures is studied. The similar problem for the rectangular thick plate has been studied in [11]. In [10,11] it was assumed that on the upper face plane of the plate the uniformly distributed normal forces act. The corresponding vibration problems are investigated in [12-14]. However, in these investigations it was supposed that the curving in the plate material structure exists only in one direction and this curving is periodic. The consistent consideration of these investigations has been made in [9].

It is known that in many cases the periodical curving in the structure of composites can exist in the two reciprocally-perpendicular directions. For example, such curving is observed in the structure of the woven-textile composites. Therefore the investigations of the corresponding problems for the elements of constructions from composites with spatially curved structures (i.e. with the structures having the curving in the two reciprocally-perpendicular directions) have an important significance. The first attempt in this field was made in [15]. Note that in [15] the stress distribution in the rectangular thick plate is considered and it is assumed that the uniformly distributed normal forces act on the upper face plane of the plate. The influence of the spatiality of the curving of the plate material structure on this stress distribution is analysed. It should be noted that, up to now, the investigations related to the corresponding stress analyses of the vibrating plates are absent completely. Taking this situation into account in the present investigation, in the framework of the continuum theory [8,9] with the use of the exact three-dimensional equations of motion of the theory of elasticity the natural and forced vibration of the rectangular thick plate from the composite with spatially periodically curved structure are investigated.

It is assumed that on the upper face plane of the plate the uniformly distributed normal forces act and these forces periodically change with respect to the time. The natural frequency of the plate and the stress distribution in it are studied. The influence of the spatiality of the periodical curving in the plate material structure on the stress distribution in that is analysed. Numerical results are obtained by employing Finite Element Method (FEM).

2. SOME PRELIMINARY REMARKS

Consider some principal moments of the continuum theory [8,9]. We isolate a representative curved packet of the composite material, shown in Fig.1. In this figure the following notation is introduced: H is the characteristic vertical rise of the structural curve; Λ_1 and Λ_3 are the halfwavelengths of periodic curves in the directions of the Ox_1 and Ox_3 axes, respectively; ΔH is the thickness of the representative packet and $\Delta H = h_1 + h_2 + ... + h_N$, where h_i $(1 \le i \le N)$ is the thickness of the i.th layer in the representative curved packed; $h' = \max\{h_1, h_2, ..., h_N\}$.

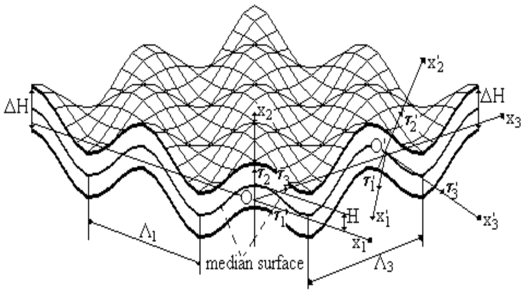

Figure 1. The geometry of the representative spatially periodically curved layer

We associate the cartesian system of coordinates $Ox_1x_2x_3$ with this packet (Fig.1). We denote orth. vectors of this system by $\boldsymbol{\tau}_i$ $(i = 1, 2, 3)$. We also introduce a local coordinate system $O'x'_1x'_2x'_3$ with the origin on the median surface of the isolated representative packet; the axis $O'x'_2$ is directed along the normal vector $\boldsymbol{\tau}'_2$, and the axes $O'x'_1$ and $O'x'_3$ along the tangential vectors $\boldsymbol{\tau}'_1$ and $\boldsymbol{\tau}'_3$ to this surface (Fig.1). The vectors $\boldsymbol{\tau}'_1$, $\boldsymbol{\tau}'_2$ and $\boldsymbol{\tau}'_3$ are determined below.

The equations of the median surface of the selected representative packet we take as follows:

$$x_2 = F(x_1, x_3) = \varepsilon f(x_1, x_3) \tag{1}$$

where ε is a parameter which is introduced for characterizing the degree of the curving; the actual determination of this parameter will be given for each specified function $F(x_1, x_3)$ in equation (1). We suppose that the function F and its first-order partial derivatives are continuous.

We assume that the elasticity relations at the chosen point of the median surface in the local coordinate system $O'x'_1x'_2x'_3$ are the following ones for the orthotropic body, with principal axes $O'x'_1$, $O'x'_2$ and $O'x'_3$.

$$\sigma_i = A^0_{ij}\varepsilon_j \quad ; \quad i, j = 1, 2, 3$$

$$\sigma_4 = 2A^0_{44}\varepsilon_4 \quad , \; \sigma_5 = 2A^0_{55}\varepsilon_5 \; , \; \sigma_6 = 2A^0_{66}\varepsilon_6 \tag{2}$$

where

$$\sigma_i = \sigma_{ii} \quad , \quad i = 1, 2, 3 \quad , \sigma_4 = \sigma_{23} \, , \, \sigma_5 = \sigma_{13} \, , \, \sigma_6 = \sigma_{12} \, ,$$

$$\varepsilon_4 = \varepsilon_{23} \quad , \; \varepsilon_5 = \varepsilon_{13} \, , \; \varepsilon_6 = \varepsilon_{12} \tag{3}$$

In (2) and (3) the conventional notation is used.

The above-stated means that in the ideal (uncurved) case the composite material is orthotrophic with principal axes $O'x'_1$, $O'x'_2$ and $O'x'_3$. Let us formulate the elasticity relations for the arbitrarily curved representative packet in the global coordinate system $Ox_1x_2x_3$. For this purpose we first determine the cosines $\ell_{ij} = \boldsymbol{\tau}'_i\boldsymbol{\tau}_j$ of the angles between the axes $O'x'_i$ and Ox_j and we rewrite equation (1) in the vectorial form

$\mathbf{r} = x_1\boldsymbol{\tau}_1 + F(x_1, x_3)\boldsymbol{\tau}_2 + x_3\boldsymbol{\tau}_3$ (4)

From equation (4) and from the relations $\boldsymbol{\tau}'_2 = (\partial\mathbf{r}/\partial x_3 \times \partial\mathbf{r}/\partial x_1)/|(\partial\mathbf{r}/\partial x_3 \times \partial\mathbf{r}/\partial x_1)|$, $\boldsymbol{\tau}''_1 = (\partial\mathbf{r}/\partial x_1)/|\partial\mathbf{r}/\partial x_1|$, $\boldsymbol{\tau}''_3 = \boldsymbol{\tau}''_1 \times \boldsymbol{\tau}'_2$ the triad of vectors $\boldsymbol{\tau}''_1$, $\boldsymbol{\tau}'_2$, $\boldsymbol{\tau}''_3$ are obtained. After rotating this triad about $\boldsymbol{\tau}'_2$ by an angle $\Psi/2$ where $Sin\Psi = F_{,1}F_{,3}V_1V_3$, we obtain the new triad of vectors $\boldsymbol{\tau}'_1$, $\boldsymbol{\tau}'_2$, $\boldsymbol{\tau}'_3$, the expressions of which are

$\boldsymbol{\tau}'_i = \ell_{ij}\boldsymbol{\tau}_j \quad , \quad i, j = 1, 2, 3$ (5)

where

$\ell_{11} = g_1V_1 + F_{,1}F_{,3}g_2V_1V_2 \quad , \quad \ell_{12} = F_{,1}g_1V_1 - F_{,3}g_2V_1V_2$

$\ell_{13} = -(1 + F_{,1}^2)g_2V_1V_2 \quad , \quad \ell_{21} = -F_{,1}V_2 \quad , \quad \ell_{22} = V_2 \quad , \quad \ell_{23} = -V_2F_{,3}$

$\ell_{31} = g_2V_1 - F_{,1}F_{,3}g_1V_1V_2 \quad , \quad \ell_{32} = F_{,1}g_2V_1 + F_{,3}g_1V_1V_2 \quad , \quad \ell_{33} = (1 + F_{,1}^2)g_1V_1V_2$ (6)

In (6) the following notation is used

$$F_{,1} = \frac{\partial F}{\partial x_1} \ , \ F_{,3} = \frac{\partial F}{\partial x_3}$$

$$V_1 = (1 + (F_{,1})^2)^{-\frac{1}{2}} \ , \ V_2 = (1 + (F_{,1})^2 + (F_{,3})^2)^{-\frac{1}{2}} \ , \ V_3 = (1 + (F_{,3})^2)^{-\frac{1}{2}}$$

$$g_1 = Cos\left(\frac{\Psi}{2}\right) \ , \ g_2 = Sin\left(\frac{\Psi}{2}\right), \ \Psi = \arcsin(F_{,1}F_{,3}V_1V_3) \qquad 7)$$

Using the well-known transformation formulas for the elastic constants under the rotation of the coordinate system, we obtain the following expressions for the normalized elastic constants of the arbitrarily curved representative packet in the global coordinate system $Ox_1x_2x_3$.

$A_{sp} = A_{nm}^0 q_{sn} q_{pm} \qquad (m; n; s; p = 1, 2, 3, 4, 5, 6)$ (8)

where A_{mn}^0 are the normalized elasticity constants of the considered composite material in the case where its structure is ideal (uncurved), i.e. the constants which enter the relations (2); q_{sm} denote the following expressions.

$q_{ij} = \ell_{ij}^2$ for $i, j = 1, 2, 3$; $q_{i4} = 2\ell_{i2}\ell_{i3}$

$q_{i5} = 2\ell_{i3}\ell_{i1}$, $q_{i6} = 2\ell_{i2}\ell_{i1}$ for $i, j = 1, 2, 3$

$q_{4i} = \ell_{3i}\ell_{2i}$, $q_{5i} = \ell_{3i}\ell_{1i}$, $q_{6i} = \ell_{2i}\ell_{1i}$, for $i, j = 1, 2, 3$

$q_{44} = \ell_{33}\ell_{22} + \ell_{32}\ell_{23}$, $q_{45} = \ell_{33}\ell_{21} + \ell_{31}\ell_{23}$, $q_{46} = \ell_{31}\ell_{22} + \ell_{32}\ell_{21}$

$q_{54} = \ell_{33}\ell_{12} + \ell_{32}\ell_{13}$, $q_{55} = \ell_{33}\ell_{11} + \ell_{31}\ell_{13}$, $q_{56} = \ell_{31}\ell_{12} + \ell_{32}\ell_{11}$

$q_{64} = \ell_{13}\ell_{22} + \ell_{12}\ell_{23}$, $q_{65} = \ell_{13}\ell_{21} + \ell_{11}\ell_{23}$, $q_{66} = \ell_{11}\ell_{22} + \ell_{12}\ell_{21}$ (9)

Thus, we obtain the following elasticity relations in the global coordinates x_i

$\sigma_i = A_{ij}\varepsilon_j \quad , \quad i; j = 1, 2, 3, 4, 5, 6$ (10)

In (10) the notation (3) is used. Note that the elasticity relations (8)-(10) are obtained without any restrictions on the curving parameters. According to (6)-(10), we can write the following relations for A_{ij} through the constants A_{ij}^0 and the function $F(x_1, x_3)$

$A_{ij} = A_{ij}(A_{nm}^0, F(x_1, x_3))$ (11)

The explicit form of the functions (11) has cumbersome expressions and therefore we don't give them here. Note also that these expressions can be established easily by the use of (6)-(9).

Thus with the above–stated we exhaust the consideration of the continuum theory [8,9], i.e. the establishing of the relations (10) or the determination of the functions A_{ij} (11). The problems considered in the present paper are investigated just in the framework of the constitutive relations (10), (11).

3. FORMULATION OF THE PROBLEM AND METHOD OF SOLUTION

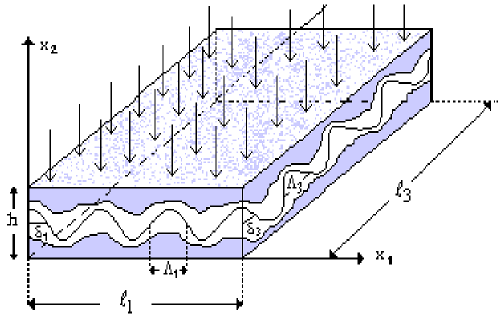

Figure 2. The geometry of the plate

Consider rectangular plate with the geometry shown in Fig.2 and assume that this plate is fabricated from the composite material with the spatially curved structure. With the plate we associate the cartesian system of coordinates and suppose that the plate occupies the region

$$\Omega = \{0 \le x_1 \le \ell_1 \ , \ 0 \le x_2 \le h \ , \ 0 \le x_3 \le \ell_3\} \tag{12}$$

Assume that the plate ends in the Ox_1 axis are clamped, but in the Ox_3 axis are simply supported. Moreover, assume that the normal harmonic forces with intensity $pe^{i\omega t}$ act on the upper face plane of the plate.

Investigate the forced vibration of this plate in the framework of the above-stated assumptions. According to the well-known consideration, the components of the stresses σ_{ij} and strain ε_{ij} tensors, and the component u_i $(i,j=1,2,3)$ of the displacement vector were presented previously as $\{\sigma_{ij},\varepsilon_{ij},u_i\}=\{\bar{\sigma}_{ij},\bar{\varepsilon}_{ij},\bar{u}_i\}\exp(i\omega t)$ and the equations were obtained for $\bar{\sigma}_{ij},\bar{\varepsilon}_{ij},\bar{u}$. In the future, we omit the bar over these symbols. Thus, for investigation of the vibration problems we have the following governing field equations.

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho\omega^2 u_i = 0 \quad , \quad \varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{13}$$

It is necessary to add to these equations the constitutive relations (10).

For the problem considered the following conditions are satisfied

$$u_1\big|_{x_1=0,\ell_1} = u_2\big|_{x_1=0,\ell_1} = u_3\big|_{x_1=0,\ell_1} = 0 \ , \quad u_2\big|_{x_3=0,\ell_3} = 0 \ , \quad \sigma_{33}\big|_{x_3=0,l_3} = 0$$

$$\sigma_{23}\big|_{x_2=0} = \sigma_{12}\big|_{x_2=0} = \sigma_{22}\big|_{x_2=0} = 0 \ , \quad \sigma_{12}\big|_{x_2=h} = \sigma_{32}\big|_{x_2=h} = 0 \ , \quad \sigma_{22}\big|_{x_2=h} = p \tag{14}$$

To solve the problem (10), (13) and (14) we employ the three-dimensional Finite Element Modelling and introduce the following functional

$$\Pi = \frac{1}{2}\iiint\limits_{\Omega} \sigma_{ij}\varepsilon_{ij} dx_1 dx_2 dx_3 - \iint\limits_{S_p} p_i u_i dx_1 dx_3 - \frac{1}{2}\iiint\limits_{\Omega} \rho\omega^2 u_i u_i dx_1 dx_2 dx_3 \qquad (15)$$

The region Ω and the plate surface S_p on which the forces act is divided into eight-node rectangular brick elements and doing standard Ritz technique operations the equation

$$(\mathbf{K} - \omega^2\mathbf{M})\overline{\mathbf{a}} = \mathbf{r} \qquad (16)$$

are obtained. In (16) $\mathbf{M}$ is a mass matrix

$$\mathbf{M} = \iiint\limits_{\Omega} \rho \mathbf{N}^{\mathrm{T}}\mathbf{N} dx_1 dx_2 dx_3 \qquad (17)$$

where $\mathbf{N}$ is a vector contains shape functions for selected three-dimensional finite elements and ρ is the volumetric density of the plate material. $\mathbf{K}$ is the stiffness matrix

$$\mathbf{K} = \sum_{K=1}^{M} \mathbf{K}^K \qquad (18)$$

where

$$\mathbf{K}^K = \iiint\limits_{\Omega_K} (\mathbf{B}^K)^T \mathbf{D}^K \mathbf{B}^K dx_1 dx_2 dx_3 \qquad (19)$$

$$(\mathbf{B}^K)^T = \begin{pmatrix} \frac{\partial N}{\partial x_1} & 0 & 0 & \frac{\partial N}{\partial x_2} & 0 & \frac{\partial N}{\partial x_3} \\ 0 & \frac{\partial N}{\partial x_2} & 0 & \frac{\partial N}{\partial x_1} & \frac{\partial N}{\partial x_3} & 0 \\ 0 & 0 & \frac{\partial N}{\partial x_3} & 0 & \frac{\partial N}{\partial x_2} & \frac{\partial N}{\partial x_1} \end{pmatrix} \qquad (20)$$

$$\mathbf{D}^{(K)} = (A_{ij}(x_1, x_3))\Big|_{x_1, x_3 \in \Omega_K} \qquad (21)$$

In (16) $\mathbf{a}$ is a vector whose components are the values of displacement at the selected nodes, $\mathbf{r}$ has the components of the force at each node and $A_{ij}(x_1, x_3)$ are determined by (10). Note that all programs in the framework of which the equation (16) is solved and the numerical investigations are carried out, have been composed by the authors in the FTN77. Accounting the symmetry of the problem about $x_1/\ell_1 = 1/2$ and $x_3/\ell_3 = 1/2$ all numerical investigations are made in the region $\{0 \le x_1/\ell_1 \le 1/2\ ,\ 0 \le x_2 \le h\ ,\ 0 \le x_3/\ell_3 \le 1/2\}$, which in turn give 2304 eight-node rectangular brick elements. In this case 24, 4 and 24 elements are arranged in the directions of the Ox_1, Ox_2 and Ox_3 axes respectively.

4. NUMERICAL RESULTS AND DISCUSSIONS

We assume that the plate material consists of the alternating layers of two isotropic, homogeneous materials and these layers are located in the Ox_1x_3 plane. We introduce the following notation for matrix and reinforcing layers: E_1, E_2 are the Young's moduli; ν_1, ν_2 are the Poisson's ratios, $\lambda_k = \nu_k E_k/(1+\nu_k)(1-2\nu_k)$ and $\mu_k = E_k/2(1+\nu_k)$ $(k = 1, 2)$ are the Lame's constants, η_1, η_2 are the concentrations of the constituents respectively. According to [16], in the considered case the constants A_{ij}^0, which enter (2) are determined by the following expressions.

$$A_{66}^0 = A_{44}^0 = \frac{\mu_1\mu_2}{\mu_1\eta_2 + \mu_2\eta_1}\ , \quad A_{55}^0 = \mu_1\eta_1 + \mu_2\eta_2\ ,\ \frac{1}{2}(A_{11}^0 - A_{13}^0) = \mu_1\eta_1 + \mu_2\eta_2$$

$$A_{23}^0 = A_{12}^0 = \lambda_1\eta_1 + \lambda_2\eta_2 - \eta_1\eta_2(\lambda_1 - \lambda_2)\frac{(\lambda_1 + 2\mu_1) - (\lambda_2 + 2\mu_2)}{(\lambda_1 + 2\mu_1)\eta_2 + (\lambda_2 + 2\mu_2)\eta_2}$$

$$\frac{1}{2}(A_{11}^0 + A_{13}^0) = (\lambda_1 + \mu_1)\eta_1 + (\lambda_2 + \mu_2)\eta_2 - \eta_1\eta_2\frac{(\lambda_1 - \lambda_2)^2}{(\lambda_1 + 2\mu_1)\eta_2 + (\lambda_2 + 2\mu_2)\eta_2}$$

$$A_{22}^0 = (\lambda_1 + 2\mu_1)\eta_1 + (\lambda_2 + 2\mu_2)\eta_2 - \eta_1\eta_2\frac{((\lambda_1 + 2\mu_1) - (\lambda_2 + 2\mu_2))^2}{(\lambda_1 + 2\mu_1)\eta_2 + (\lambda_2 + 2\mu_2)\eta_2}$$

$$A_{23}^0 = A_{12}^0, \quad A_{33}^0 = A_{11}^0, \quad A_{11}^0 - A_{13}^0 = 2A_{55}^0 \tag{22}$$

We assume that the curving in the plate material structure is periodic and its form, i.e. the function (1) is given by the following equation.

$$x_2 = F(x_1, x_3) = HSin(\frac{\ell_1}{\Lambda_1}\frac{\pi x_1}{\ell_1} + \delta_1)Sin(\frac{\ell_3}{\Lambda_3}\frac{\pi x_3}{\ell_3} + \delta_3)$$

$$= \varepsilon\Lambda_1 Sin(\frac{\ell_1}{\Lambda_1}\frac{\pi x_1}{\ell_1} + \delta_1)Sin(\frac{\ell_3}{\Lambda_3}\frac{\pi x_3}{\ell_3} + \delta_3), \qquad \varepsilon = \frac{H}{\Lambda_1} \tag{23}$$

where Λ_1 and Λ_2 are the half wavelength of the curving form along the Ox_1 and Ox_3 axes respectively; ε estimates the degree of curving (if $\varepsilon = 0$, there is no curving); $\delta_1(\delta_3)$ characterizes the distance by which the origin of the system of coordinate $Ox_1x_2x_3$ shifts along the $Ox_1(Ox_3)$ axis relative to the first "nodal" point of curving form (Fig.2).Thus, through the expressions (23), (22), (7), (9) and (8) we determine the functions $A_{ij}(x_1, x_3)$ which enter (10) and (21).

Now we consider the numerical results obtained in the framework of the above-stated approach. Introduce the dimensionless frequency $\overline{\omega}$ by $\overline{\omega}^2 = \omega^2\rho\ell_1^2 / A_{22}^0$ and dimensionless parameters $\gamma_1 = \ell_1/\Lambda_1$ and $\gamma_3 = \ell_3/\Lambda_3$. For simplicity assume that $\nu_1 = \nu_2 = 0.3$; $h/\ell_1 = 0.1$, $\eta_1 = \eta_2 = 0.5$, $\delta_1 = \delta_3 = \pi/2$.

Later, we omit the bar above in $\overline{\omega}$ and consider the fundamental frequency (denote it by ω_*) determined from the requirement $u_2 \to \infty$ under $\omega \to \omega_*$, where the values of u_2 are determined from the equation (16).

Table 1. The values of the fundamental frequency ω_*^2 for various $\gamma_1 = \ell_1/\Lambda_1$ and $\gamma_3 = \ell_3/\Lambda_3$ under $\ell_1/\ell_3 = 1$, $E^{(2)}/E^{(1)} = 50$, $\varepsilon = 0.1$

γ_1	γ_3					
	0	2	4	6	8	10
2	2.523	2.607	2.635	2.693	2.831	3.002
4	2.491	2.588	2.569	2.548	2.581	2.680
6	2.532	2.603	2.580	2.546	2.538	2.580
8	2.561	2.621	2.602	2.571	2.550	2.562
10	2.592	2.645	2.631	2.606	2.585	2.586

Table 1 shows the values of ω_*^2 for various γ_1 and γ_3 under $\ell_1/\ell_3 = 1$, $E_2/E_1 = 50$, $\varepsilon = 0.1$. Note that for the case $\varepsilon = 0.0$ we obtain that $\omega_*^2 = 2.634$. It follows from these data that under unidirected curving (i.e. under $\gamma_3 = 0$) the dependence between ω_*^2 and γ_1 is non monotonic. As an influence of the existence of the curving in the direction of the Ox_3 axis (i.e. the cases where $\gamma_3 \neq 0$) values of ω_*^2 increase monotonically with γ_3 under $\gamma_1 = 2, 4$. However, in the cases where $\gamma_1 = 6,\ 8,\ 10$ the dependence between ω_*^2 and γ_3 is non monotonical. Table 1 shows that minimal values for ω_*^2 are obtained in the cases where $\gamma_1 = \gamma_3 \geq 6$. Note also that the values of ω_*^2 obtained in the case $\gamma_3 = 0$ very nearly correspond to those given in [9]. The data given in [9] are obtained by employing the semi-analytical FEM modelling with the use of the rectangular Lagrange family quadratic elements. This closeness of the corresponding results presented here and in [9] guarantees the trustiness of the applied solution technique.

Table 2. The values of the fundamental frequency ω_*^2 for various $E^{(2)}/E^{(1)}$ and ℓ_3/ℓ_1 under $\varepsilon = 0.1$, $\gamma_1 = \gamma_3 = 8$

E_2/E_1	ℓ_3/ℓ_1			
	1	1.5	2	4
5	0.776	0.599	0.548	0.504
10	1.117	0.866	0.793	0.731
50	2.550	1.970	1.796	1.641
100	3.469	2.639	2.387	2.154

Table 2 shows the values of ω_*^2 for various E_2/E_1 and ℓ_3/ℓ_1 under $\varepsilon = 0.1$, $\gamma_1 = \gamma_3 = 8$. These results show that ω_*^2 increases (decreases) monotonically with E_2/E_1 (ℓ_3/ℓ_1). This statement agrees with the well-known mechanical consideration.

Now we consider the stress distribution in the plate under forced vibration and analyze the stress σ_{22} in the cases where $\omega^2 << \omega_*^2$. We investigate the distribution of the stress σ_{22} with respect to x_1/ℓ_1 in the middle plane (i.e. in the $x_2 = h/2$ (Fig.2)) for the section $x_3 = \ell_3/2$. Note that adhesion strength of the plate material depends mainly on the stress σ_{22} and the values of this stress can be calculated with high accuracy by employing the three-dimensional exact equations of the theory of elasticity. Therefore, the studying of the stress σ_{22} has an important significance.

In Fig.3 the graphs of the dependence between σ_{22}/p and x_1/ℓ_1 are given. These graphs are constructed in the case where $\varepsilon = 0.1$, $\gamma_1 = 8$, $\omega^2 = 0.7$, $E_2/E_1 = 50$, $\ell_1/\ell_3 = 1$ for various γ_3. It follows from these graphs that the absolute values of the local maximum in the distributions of σ_{22}, i.e. the values of the $loc.\max\left\| \sigma_{22}|_{\varepsilon=0} \right| - \left| \sigma_{22}|_{\varepsilon=0.1} \right\|$ which arise as a result of the curving, increase monotonically with γ_3.

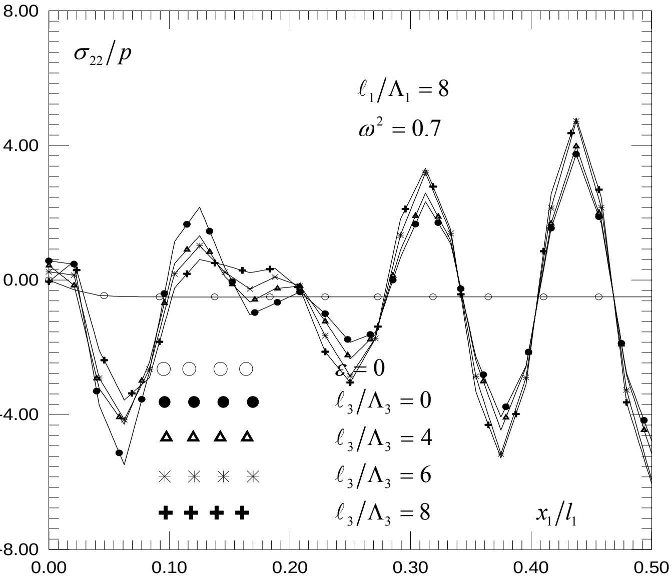

Figure 3. The graphs of the dependence between σ_{22}/p and x_1/ℓ_1 for various $\gamma_3 = \ell_3/\Lambda_3$ under $\gamma_1 = 8$, $x_2 = h/2$, $E^{(2)}/E^{(1)} = 50$, $\omega^2 = 0.7$, $\varepsilon = 0.1$, $\ell_1/\ell_3 = 1$, $x_3 = \ell_3/2$

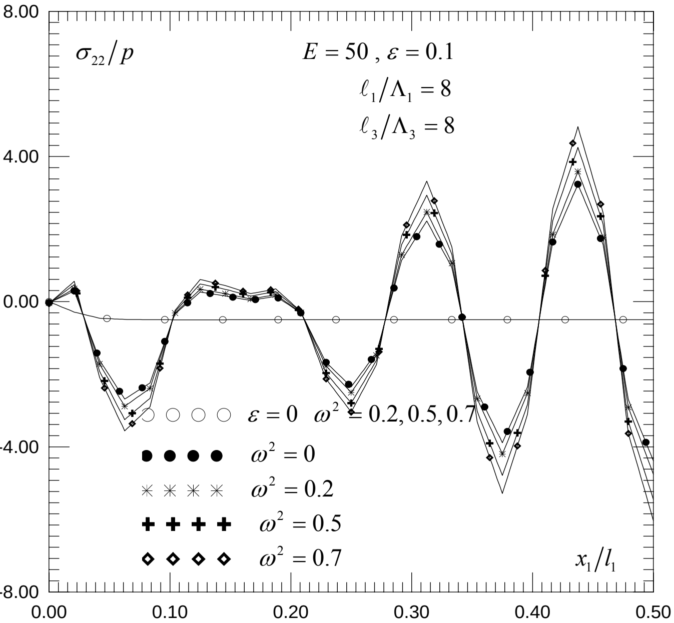

Figure 4. The graphs of the dependence between σ_{22}/p and x_1/ℓ_1 for various ω^2 under $\ell_1/\ell_3 = 1$, $\gamma_1 = \gamma_3 = 8$, $\varepsilon = 0.1$, $E^{(2)}/E^{(1)} = 50$, $x_2 = h/2$, $x_3 = \ell_3/2$

The graphs of the dependence between σ_{22}/p and x_1/ℓ_1 constructed for various ω^2 under $\ell_1/\ell_3 = 1$, $\gamma_1 = \gamma_3 = 8$, $\varepsilon = 0.1$, $E_2/E_1 = 50$ and given in Fig.4 show that the absolute values of the local maximum of the σ_{22} increase monotonically with ω^2. Note that similar results are obtained for the other problem parameters.

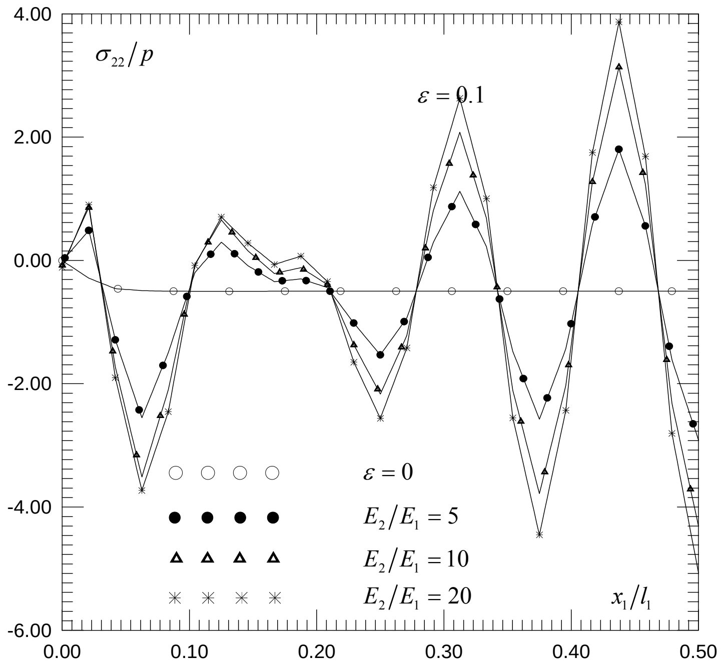

Figure 5. The graphs of the dependence between σ_{22}/p and x_1/ℓ_1 for various E_2/E_1 under $\omega^2 = 0.1\text{x}\omega_*^2$, $\ell_1/\ell_3 = 1$, $\varepsilon = 0.1$, $\gamma_1 = \gamma_3 = 8$, $x_2 = h/2$, $x_3 = \ell_3/2$

Fig.5 also shows the graphs of the dependence between σ_{22}/p and x_1/ℓ_1. However, these graphs are constructed for various E_2/E_1 and for each E_2/E_1 the values of ω^2 are selected as $\omega^2 = 0.1\times\omega_*^2$, where ω_*^2 is the corresponding fundamental frequency. Furthermore, it was assumed that $\ell_1/\ell_3 = 1$, $\varepsilon = 0.1$, $\gamma_1 = \gamma_3 = 8$. The comparison of these graphs shows that the absolute values of σ_{22} increase monotonically with E_2/E_1.

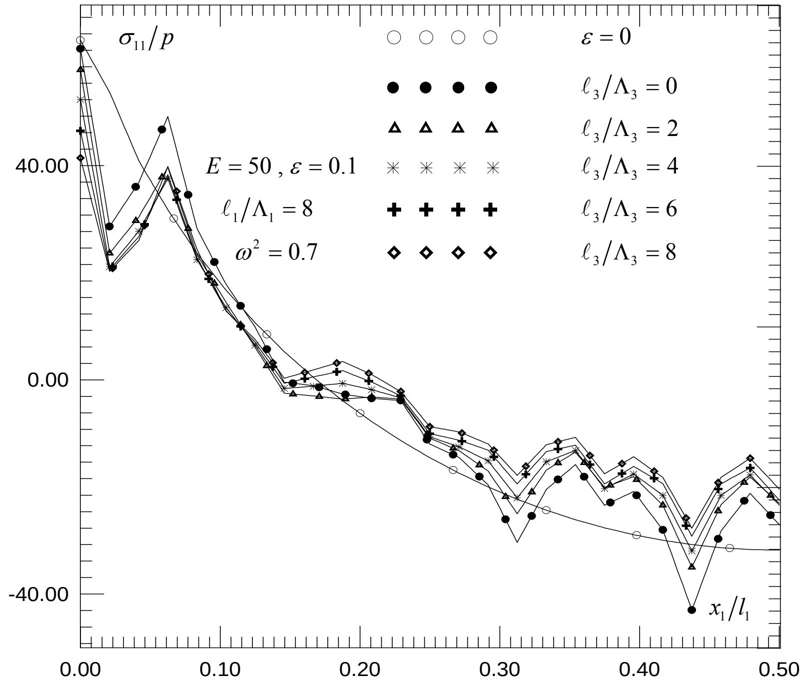

Figure 6. The graphs of the dependence between σ_{11}/p and x_1/ℓ_1 for various γ_3 under $\omega^2 = 0.7$, $\gamma_1 = 8$, $\varepsilon = 0.1$, $E^{(2)}/E^{(1)} = 50$, $x_2 = h$, $x_3 = \ell_3/2$

Now we consider some numerical results related to the distribution of the normal stress σ_{11} with respect to x_1/ℓ for $x_2 = h$, $x_3/\ell_3 = 0.5$ under various problem parameters. Consider the graphs given in Fig.6, which show this distribution for various γ_3 with $\omega^2 = 0.7$, $\gamma_1 = 8$, $\varepsilon = 0.1$, $E^{(2)}/E^{(1)} = 50$. It follows from these results that under $x/\ell_1 \geq 0.2$ the graphs lift up totally with γ_3 . However, the absolute values of local maximum of perturbations, i.e. of the $loc.\max\left\|\sigma_{11}\right|_{\varepsilon=0}\left| - \right|\sigma_{11}\left|_{\varepsilon=0.1}\right\|$, increase with γ_3 .

The graphs illustrated in Fig.7 show also the mentioned distribution of the stress σ_{11} for various ω^2 under $E^{(2)}/E^{(1)} = 50$, $\gamma_1 = 8$, $\gamma_3 = 8$, $\varepsilon = 0.1$. These graphs show that for the considered ω^2 the values of the $loc.\max\left\|\sigma_{11}\right|_{\varepsilon=0}\left| - \right|\sigma_{11}\left|_{\varepsilon=0.1}\right\|$ decrease, but the values of $\left|\sigma_{11}\right|_{\varepsilon=0.1}$ increase with ω^2 .

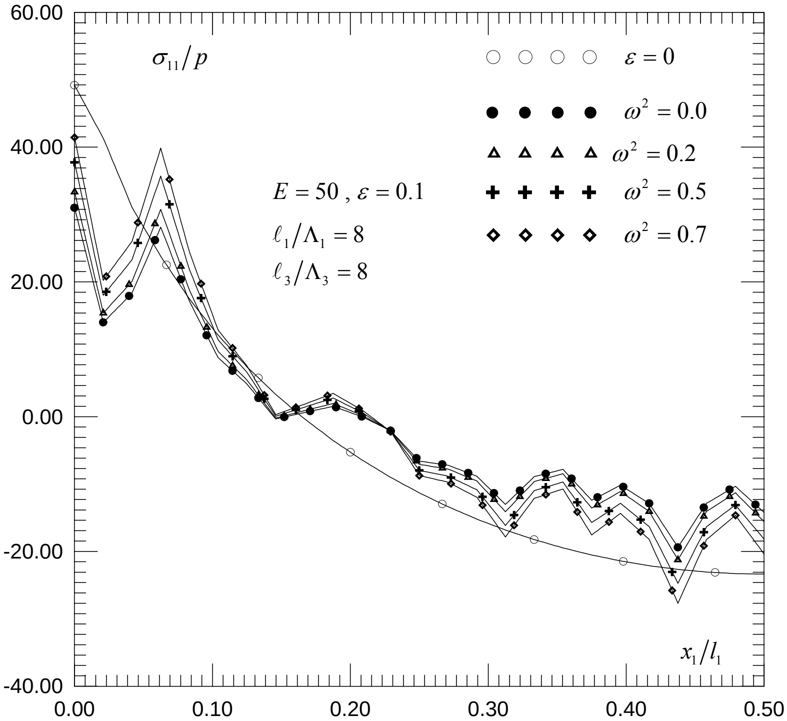

Figure 7. The graphs of the dependence between σ_{11}/p and x_1/ℓ_1 for various ω^2 under $\gamma_1 = \gamma_3 = 8$, $E^{(2)}/E^{(1)} = 50$, $\varepsilon = 0.1$, $x_2 = h$, $x_3 = \ell_3/2$

5. CONCLUSIONS

The following conclusions can be drawn:

In the framework of continuum approach [8,9] and with the use of the three-dimensional exact equations of motion of the theory of elasticity the natural and forced vibration of the plate fabricated from the composite material with spatially periodically curved structure are investigated. It was assumed that the force that acts on the upper face plane is of the plate the uniformly distributed force which is harmonic in time. Moreover, it was assumed that the plate is clamped at the two opposite ends and is simply supported in the other two opposite ends. By employing the three-dimensional FEM modelling the numerical results related to the fundamental frequency and to the distribution of the normal stresses σ_{22} , σ_{11} are presented, where σ_{22} acts along the thickness of the plate, but the stress σ_{11} acts along the length of the plate (Fig.2).

According to the numerical results, we can in particular, conclude that the spatiality of the curving in the plate material structure causes the absolute values of the local maximum of the perturbation of the stress σ_{22} , i.e. of the $loc.\max \left\| \sigma_{22}\big|_{\varepsilon=0} \right| - \left| \sigma_{22}\big|_{\varepsilon=0.1} \right\|$ to increase.

Note that these perturbations arise as a result of the existence of the curving of the normal stress σ_{22} acting along the thickness of the plate and therefore under prognostication of the adhesion strength of the plate material these perturbations must be taken into account.

Consequently, the obtained numerical results and the investigations carried out in the present paper have a great significance for engineering applications of the composites with spatially periodical curved structure.

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