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GREEN'S FUNCTION OF DISCONTINUOS BOUNDARY VALUE PROBLEM WITH EIGENPARAMETER IN BOUNDARY CONDITIONS

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ABSTRACT

We constructed Green's function solution of discontinuous Sturm-Liouville problems with eigenparameter in boundary conditions.
Keywords: Discontinuous Sturm–Liouville problems, eigenparameter, Green's function, eigenvalue, eigenfunction.
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SINIR KOŞULUNDA ÖZ PARAMETRE BULUNAN SÜREKSİZ SINIR DEĞER PROBLEMİNİN GREEN FONKSİYONU

ÖZET

Bu makalede sınır koşullarında özparametre bulunan, süreksiz Sturm-Liouville problemleri için Green fonksiyonu inşa edilmiştir.
Anahtar Sözcükler: Süreksiz Sturm-Liouville problemleri, özparametre, Green fonksiyonu, özdeğer, özfonksiyon.

1. INTRODUCTION

In this paper, we establish the Green's function for Sturm-Liouville equation

$$- y''(x) + q(x)y(x) = \lambda y(x) \tag{1.1}$$

on the interval $\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$, with the eigenparameter-dependent boundary conditions

$$\sqrt{\lambda}\, y(0) + y'(0) = 0 \quad , \tag{1.2}$$

$$\lambda y(\pi) + y'(\pi) = 0 \tag{1.3}$$

and the transmission conditions

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$$y\left(\frac{\pi}{2}+0\right)-\delta y\left(\frac{\pi}{2}-0\right)=0\,, \tag{1.4}$$

$$y'\left(\frac{\pi}{2}-0\right)-\delta y'\left(\frac{\pi}{2}+0\right)=0\,, \tag{1.5}$$

where the real valued function $q(x)$ is continuous on $\left[0,\frac{\pi}{2}\right)\bigcup\left(\frac{\pi}{2},\pi\right]$ having the finite limits $q\left(\frac{\pi}{2}+0\right)=\lim\limits_{x\to\frac{\pi}{2}+0} q(x)$ and $q\left(\frac{\pi}{2}-0\right)=\lim\limits_{x\to\frac{\pi}{2}-0} q(x)$, λ is a real eigenparameter and, $\delta\neq 0$ is an arbitrary real number.

Many topics in mathematical physics require investigations of eigenvalues and eigenfunctions of boundary value problems. In spite of the fact that the general theory and methods of boundary value problems with continuous coefficients are highly developed, very little is known about a general character of similar problems with discontinuity. Some problems with transmission conditions which arise in mechanics, such as thermal conduction problems for a thin laminated plate, were studied in [1].

In recent years, continuous results have been obtained for the boundary value problems with eigenparameter dependent boundary conditions. Some of these results can be seen in [2, 3, 4, 5, 6, 7, 8]. In particular, [4, 6, 9, 10] contain many references to problems in physics and mechanics. Some special cases of the problem (1.1)-(1.5) arise from applications of the method of separation of variables to the varied assortment of physical problems [1, 5, 10]. It must be noted that asymptotic formulas of eigenvalues and eigenfunctions of this problem are investigated in [11].

In this paper we will consider eigenparameter dependent boundary conditions and will extend some results of the standard Sturm-Liouville problems to discontinuous cases. In particular, we will construct Green's function for the problem (1.1)-(1.5) using a method described in [12].

2. SOME BASIC SOLUTIONS ACCORDING TO TRANSMISSION CONDITIONS

We define two fundamental solutions

$$\phi(x,\lambda)=\begin{cases}\phi_1(x,\lambda) & \text{for } x\in\left[0,\frac{\pi}{2}\right)\\ \phi_2(x,\lambda) & \text{for } x\in\left(\frac{\pi}{2},\pi\right]\end{cases} \tag{2.1}$$

and

$$\chi(x,\lambda)=\begin{cases}\chi_1(x,\lambda) & \text{for } x\in\left[0,\frac{\pi}{2}\right)\\ \chi_2(x,\lambda) & \text{for } x\in\left(\frac{\pi}{2},\pi\right]\end{cases} \tag{2.2}$$

of the differential equation (1.1) which satisfy one of the boundary conditions in (1.2), (1.3) and both of the transmission conditions (1.4), (1.5) as follows.

Let $\phi_1(x,\lambda)$ be a solution of the equation (1.1) on $\left[0,\frac{\pi}{2}\right)$, satisfying the initial conditions

$$\phi_1(0,\lambda)=1 \ , \ \phi_1^{'}(0,\lambda)=-\sqrt{\lambda} \ . \tag{2.3}$$

It has been shown in [11] that the solution of (1.1) with (2.3) is unique on $\left[0,\frac{\pi}{2}\right]$. Now consider the differential equation (1.1) on $\left(\frac{\pi}{2},\pi\right]$ together with the special type initial conditions

$$y\left(\frac{\pi}{2}\right)=\delta\phi_1\left(\frac{\pi}{2}-0,\lambda\right), \tag{2.4}$$

$$y^{'}\left(\frac{\pi}{2}\right)=\frac{1}{\delta}\phi_1^{'}\left(\frac{\pi}{2}-0,\lambda\right). \tag{2.5}$$

We will prove that these initial conditions define a unique solution $y=\phi_2(x,\lambda)$, which is also an entire function of the parameter $\lambda\in C$ for each fixed $x\in\left[\frac{\pi}{2},\pi\right]$. Consider the sequence $y_n(x,\lambda),\ n=0,1,2,...$ defined by the recurrence formula

$$y_n(x)=\phi_1\left(\frac{\pi}{2}-0,\lambda\right)+\left(x-\frac{\pi}{2}\right)\phi'_1\left(\frac{\pi}{2}-0,\lambda\right)+\int_{\pi/2}^{x}\big(q(t)-\lambda\big)y_{n-1}(t,\lambda)(x-t)dt. \tag{2.6}$$

Let $Q=\max\limits_{x\in\left(\frac{\pi}{2},\pi\right]}|q(x)|$. It is obvious that for each positive, real number $N>0$ there is $K>0$ such that $\left|\phi_1^{'}\left(\frac{\pi}{2}-0,\lambda\right)\left(x-\frac{\pi}{2}\right)\right|\le K$ for all $x\in\left[\frac{\pi}{2},\pi\right]$ and $|\lambda|<N$.

Thus,

$$|y_1(x)-y_0(x)|\le\int_{\pi/2}^{x}(Q+N)K(x-t)dt=\frac{1}{2}(Q+N)K\left(x-\frac{\pi}{2}\right)^2 . \tag{2.7}$$

By induction, it follows from (2.7) that

$$|y_n(x)-y_{n-1}(x)|\le(Q+N)\int_{\pi/2}^{x}|y_{n-1}(t,\lambda)-y_{n-2}(t,\lambda)|(x-t)dt \ \text{ for } \ n=2,3,... \tag{2.8}$$

$$\left|y_n(x)-y_{n-1}(x)\right| \le \frac{K(Q+N)^n\left(x-\frac{\pi}{2}\right)^{n+1}}{(n+1)!} \quad \text{for} \quad n=2,3,.. . \tag{2.9}$$

Hence, the series

$$\phi_2(x) = y_0(x) + \sum_{n=1}^{\infty}\left(y_n(x) - y_{n-1}(x)\right) \tag{2.10}$$

converges uniformly for values of λ satisfying $|\lambda| \le N$ in the interval $\frac{\pi}{2} < x \le \pi$ for $n \ge 2$. Moreover, we can obtain the following equations by differentiating the equation (2.6)

$$\left(y'_n(x) - y'_{n-1}(x)\right) = \int_{\pi/2}^{x}(q(t)-\lambda)\left(y_{n-1}(t,\lambda) - y_{n-2}(t,\lambda)\right)dt \quad , \tag{2.11}$$

$$\left(y''_n(x) - y''_{n-1}(x)\right) = (q(x)-\lambda)\left(y_{n-1}(x,\lambda) - y_{n-2}(x,\lambda)\right) \quad . \tag{2.12}$$

By virtue of (2.9) each of the series

$$\sum_{n=1}^{\infty}\int_{\pi/2}^{x}(q(t)-\lambda)\left(y_{n-1}(t,\lambda) - y_{n-2}(t,\lambda)\right)dt$$

and

$$\sum_{n=1}^{\infty}(q(x)-\lambda)\left(y_{n-1}(x,\lambda) - y_{n-2}(x,\lambda)\right)$$

converge uniformly for $|\lambda| \le N$ on the interval $\left(\frac{\pi}{2},\pi\right]$.

Hence, it follows from (2.11) and (2.12) that the differentiated series

$$\sum_{n=1}^{\infty}\left(y'_{n-1}(x,\lambda) - y'_{n-2}(x,\lambda)\right) \quad \text{and} \quad \sum_{n=1}^{\infty}\left(y''_{n-1}(x,\lambda) - y''_{n-2}(x,\lambda)\right)$$

also converge uniformly in x on the interval $\left(\frac{\pi}{2},\pi\right]$. Now taking (2.10) and (2.12) into account, we have

$$\phi_2''(x,\lambda)=\sum_{n=1}^{\infty}\left(y''_n(x,\lambda)-y''_{n-1}(x,\lambda)\right)$$

$$=(q(x)-\lambda)\sum_{n=1}^{\infty}\left(y_{n-1}(x,\lambda)-y_{n-2}(x,\lambda)\right)$$

$$=(q(x)-\lambda)\phi_2(x,\lambda).$$

Thus $\phi_2(x,\lambda)$ satisfies the differential equation (1.1) on the interval $\left(\frac{\pi}{2},\pi\right]$ and the initial conditions (2.4) and (2.5). Hence, the function $\phi(x,\lambda)$ defined by equation (2.1) satisfies the differential equation (1.1) on $\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$, the boundary condition (1.2) and the transmission conditions (1.4) and (1.5).

We now turn to the unique solution $\chi_2(x,\lambda)$ of the equation (1.1) on $\left(\frac{\pi}{2},\pi\right]$ satisfying the initial conditions

$$\chi_2(\pi,\lambda)=-1 \text{ and } \chi_2'(\pi,\lambda)=\lambda \tag{2.13}$$

which is an entire function of λ for fixed x.

The function $\chi_1(x,\lambda)$ will be defined in terms of $\chi_2(x,\lambda)$ and by the conditions

$$y\left(\frac{\pi}{2}\right)=\frac{1}{\delta}\chi_2\left(\frac{\pi}{2}+0,\lambda\right), \tag{2.14}$$

$$y'\left(\frac{\pi}{2}\right)=\delta\chi'_2\left(\frac{\pi}{2}+0,\lambda\right). \tag{2.15}$$

Applying the same technique as in the definition of $\phi_2(x,\lambda)$ we can prove that the equation (1.1) with conditions (2.14)-(2.15) has a unique solution $\chi_1(x,\lambda)$ which is also an entire function of

λ for fixed $x\in\left[0,\frac{\pi}{2}\right)$. Thus $\chi(x,\lambda)$ satisfies the differential equation (1.1), the boundary condition (1.3) and the transmission conditions (1.4) and (1.5) on $\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$.

The Wronskian

$$\omega_i(\lambda)=W_\lambda(\phi_i,\chi_i;x)=\phi_i(x,\lambda)\chi_i'(x,\lambda)-\chi_i(x,\lambda)\phi_i'(x,\lambda) \tag{2.16}$$

are independent of x for $i=1,2$ and it can easily be shown that

$$\omega_1(\lambda)=\omega_2(\lambda)=\omega(\lambda). \tag{2.17}$$

Theorem 1: The eigenvalues of the problem (1.1)-(1.5) consist of zeros of the function $\omega(\lambda)$.
Theorem can be proved easily by the same technique as for Theorem 1 in [12].

Lemma 1: Let $\lambda = s^2$ .Then the following integral equations hold.

$$\phi_1(x) = -Sin\, sx + Cos\, sx + \frac{1}{s}\int_0^x q(\tau) Sin\, s(x-\tau)\phi_1(\tau)d\tau \tag{2.18}$$

$$\phi_2(x) = \frac{1}{\delta s} Sin\, s\left(x - \frac{\pi}{2}\right)\phi_1'\left(\frac{\pi}{2}\right) + \delta Cos\, s\left(x - \frac{\pi}{2}\right)\phi_1\left(\frac{\pi}{2}\right) + \frac{1}{s}\int_0^x q(\tau) Sin\, s(x-\tau)\phi_2(\tau)d\tau \tag{2.19}$$

Proof: Consider the solution $\phi_1(x,\lambda)$ of the differential equation (1.1)

$$-\phi_1''(x) + q(x)\phi_1(x) = s^2\phi_1(x). \tag{2.20}$$

Multiplying both sides by $Sin\, s(x-\tau)$ and then integrating we get

$$-\int_0^x \phi_1''(\tau) Sin\, s(x-\tau)d\tau - s^2\int_0^x \phi_1(\tau) Sin\, s(x-\tau)d\tau + \int_0^x q(\tau) Sin\, s(x-\tau)\phi_1(\tau)d\tau = 0$$

for $x \in \left[0, \frac{\pi}{2}\right)$. After integrating by parts twice the first integral and using the conditions in (2.3) we obtain

$$\phi_1(x) = -Sin\, sx + Cos\, sx + \frac{1}{s}\int_0^x q(\tau) Sin\, s(x-\tau)\phi_1(\tau)d\tau .$$

Similarly, performing the same calculations for $\phi_2(x)$ and using the conditions (2.4) and (2.5) we get

$$\phi_2(x) = \frac{1}{\delta s} Sin\, s\left(x - \frac{\pi}{2}\right)\phi_1'\left(\frac{\pi}{2}\right) + \delta Cos\, s\left(x - \frac{\pi}{2}\right)\phi_1\left(\frac{\pi}{2}\right) + \frac{1}{s}\int_0^x q(\tau) Sin\, s(x-\tau)\phi_2(\tau)d\tau$$

for $x \in \left(\frac{\pi}{2}, \pi\right]$.

Lemma 2: Let $\lambda = s^2$ for s being a complex number. Let $\operatorname{Im} s = t$. Then, the following asymptotic equations hold for $|\lambda| \to \infty$.

$$\phi_1(x,\lambda) = \sqrt{2} Cos\left(sx + \frac{\pi}{4}\right) + O\left(s^{-1}e^{|t|x}\right) \tag{2.21}$$

$$\phi_2(x,\lambda) = -\frac{\sqrt{2}}{\delta} Sin\, s\left(x-\frac{\pi}{2}\right) Sin\left(s\frac{\pi}{2}+\frac{\pi}{4}\right) + \sqrt{2}\delta\, Cos\, s\left(x-\frac{\pi}{2}\right) Cos\left(s\frac{\pi}{2}+\frac{\pi}{4}\right) + O\left(s^{-1}e^{|t|x}\right) \tag{2.22}$$

$$\chi_2(x,\lambda) = s\, Sin\, s(x-\pi) + O\left(e^{|t|x}\right) \tag{2.23}$$

$$\chi_1(x,\lambda) = \delta\, s\, Sin\, s\left(x-\frac{\pi}{2}\right) Cos\, s\left(\frac{\pi}{2}\right) - \frac{1}{\delta} s\; Cos\, s\left(x-\frac{\pi}{2}\right) Sin\, s\left(\frac{\pi}{2}\right) + O\left(e^{|t|x}\right) \tag{2.24}$$

Theorem 2: Let $\lambda = s^2$ and $\operatorname{Im} s = t$. The asymptotic representation of characteristic function $\omega(\lambda)$ is

$$\omega(\lambda) = \delta\sqrt{2}s^2\, Cos\, s\frac{\pi}{2} Cos\left(s\frac{\pi}{2}+\frac{\pi}{4}\right) - \frac{\sqrt{2}s^2}{\delta} Sin\, s\frac{\pi}{2} Sin\left(s\frac{\pi}{2}+\frac{\pi}{4}\right) + O\left(e^{|t|x}\right). \tag{2.25}$$

Proof: This result is a direct consequence of equation (2.16) and Lemma 2.

3. ASYMPTOTIC REPRESENTATION OF GREEN FUNCTION

Let us consider Sturm-Liouville equation

$$L(y) = -y''(x) + q(x)y(x) = -f(x) \tag{3.1}$$

together with eigenparameter-dependent boundary conditions (1.2)-(1.3) and transmission conditions (1.4)-(1.5). Here $f(x)$ is continuous function on the interval $\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$.

We can represent the general solution of (3.1) in the following form

$$Y(x,\lambda) = \begin{cases} A_1\phi_1(x,\lambda) + B_1\chi_1(x,\lambda) & for\ x\in\left[0,\frac{\pi}{2}\right) \\ A_2\phi_2(x,\lambda) + B_2\chi_2(x,\lambda) & for\ x\in\left(\frac{\pi}{2},\pi\right] \end{cases}. \tag{3.2}$$

We applied the standard method of variation of the constants to (3.2), thus, the functions $A_1(x,\lambda), B_1(x,\lambda)$ and $A_2(x,\lambda), B_2(x,\lambda)$ satisfies the linear system of equations

$$\begin{aligned} &A_1'\phi_1(x,\lambda) + B_1'\chi_1(x,\lambda) = 0, \\ &A_1'\phi_1'(x,\lambda) + B_1'\chi_1'(x,\lambda) = f(x), \end{aligned} \tag{3.3}$$

for $x\in\left[0,\frac{\pi}{2}\right)$ and

$$A_2'\phi_2(x,\lambda)+B_2'\chi_2(x,\lambda)=0,$$
$$A_2'\phi_2'(x,\lambda)+B_2'\chi_2'(x,\lambda)=f(x), \tag{3.4}$$

for $x\in\left(\frac{\pi}{2},\pi\right]$. Since λ is not an eigenvalue and

$$W_\lambda(\phi_i,\chi_i;x)\neq 0 \quad \text{for } i=1,2$$

each of the linear system in (3.3) and (3.4) has a unique solution which leads

$$A_1(x,\lambda)=\frac{1}{\omega(\lambda)}\int_x^{\pi/2} f(y)\chi_1(y,\lambda)dy+A_1 \quad x\in\left[0,\frac{\pi}{2}\right),$$

$$B_1(x,\lambda)=\frac{1}{\omega(\lambda)}\int_0^{x} f(y)\phi_1(y,\lambda)dy+B_1 \quad x\in\left[0,\frac{\pi}{2}\right),$$

$$A_2(x,\lambda)=\frac{1}{\omega(\lambda)}\int_x^{\pi} f(y)\chi_2(y,\lambda)dy+A_2 \quad x\in\left(\frac{\pi}{2},\pi\right],$$

$$B_2(x,\lambda)=\frac{1}{\omega(\lambda)}\int_{\pi/2}^{x} f(y)\phi_2(y,\lambda)dy+B_2 \quad x\in\left(\frac{\pi}{2},\pi\right].$$

Where A_1, A_2, B_1 and B_2 are arbitrary constants. Substituting these expressions to (3.2), we obtain the solution of (3.1)

$$Y(x,\lambda)=\frac{\phi_1(x,\lambda)}{\omega(\lambda)}\int_x^{\pi/2} f(y)\chi_1(y,\lambda)dy+\frac{\chi_1(x,\lambda)}{\omega(\lambda)}\int_0^{x} f(y)\phi_1(y,\lambda)dy+A_1\phi_1(x,\lambda)+B_1\chi_1(x,\lambda) \tag{3.5}$$

for $x\in\left[0,\frac{\pi}{2}\right)$ and

$$Y(x,\lambda)=\frac{\phi_2(x,\lambda)}{\omega(\lambda)}\int_x^{\pi} f(y)\chi_2(y,\lambda)dy+\frac{\chi_2(x,\lambda)}{\omega(\lambda)}\int_{\pi/2}^{x} f(y)\phi_2(y,\lambda)dy+A_2\phi_2(x,\lambda)+B_2\chi_2(x,\lambda) \tag{3.6}$$

for $x\in\left(\frac{\pi}{2},\pi\right]$.

Substituting these solutions into the eigenparameter-dependent boundary conditions (1.2)-(1.3) and the transmission conditions (1.3)-(1.4), we obtain the resolvent of the boundary value problem (1.1)-(1.5) as

$$Y(x,\lambda)=\frac{\chi(x,\lambda)}{\omega(\lambda)}\int_0^{x} f(y)\phi(y,\lambda)dy+\frac{\phi(x,\lambda)}{\omega(\lambda)}\int_x^{\pi} f(y)\chi(y,\lambda)dy \ . \tag{3.7}$$

We can now easily find the Green's function of the problem (1.1)-(1.5) from the resolvent (3.1).

Theorem 3: If the equation $L(u)=0$ has only trivial solution, then, for any function $f(x)$ which is continuous on the interval $[a,b]$, there exists a solution of the equation $Lu=f(x)$ given by

$$u(x)=\int_a^b G(x,\xi;\lambda)f(\xi)d\xi\ .$$

Proof of this theorem can be found in [13] where $G(x,\xi;\lambda)$ denotes the Greeen's function for the operator L.

From Theorem 3 we have

$$Y(x,\lambda)=\int_0^\pi G(x,y;\lambda)f(y)dy\ .$$

and from (3.7), the Green's function of the problem (1.1)-(1.5) can be represented as follows

$$G(x,y;\lambda)=\begin{cases}\dfrac{1}{\omega(\lambda)}\chi(x,\lambda)\phi(y,\lambda), & 0\le y\le x\le\pi,\ x\ne\dfrac{\pi}{2},\ y\ne\dfrac{\pi}{2}\\ \dfrac{1}{\omega(\lambda)}\chi(y,\lambda)\phi(x,\lambda), & 0\le x\le y\le\pi,\ x\ne\dfrac{\pi}{2},\ y\ne\dfrac{\pi}{2}\end{cases}\ . \tag{3.8}$$

Finally, in view of the condition (2.10) and using the asymptotic formulas in (2.21)-(2.25), we obtain the asymptotic representation of Green's function $G(x,y;\lambda)$ for $|\lambda|\to\infty$ as

$$G(x,y;\lambda)=\begin{cases} \dfrac{Cos\left(sy+\frac{\pi}{4}\right)\left(Sin\,s\left(x-\frac{\pi}{2}\right)\right)}{s\,Cos\left(s\frac{\pi}{2}+\frac{\pi}{4}\right)}+O\left(s^{-2}e^{|t|(y-x)}\right) & 0\le y\le x<\frac{\pi}{2} \\ \dfrac{Sin\,s(x-\pi)Cos\left(sy+\frac{\pi}{4}\right)}{\delta s\,Cos\,s\frac{\pi}{2}Cos\left(s\frac{\pi}{2}+\frac{\pi}{4}\right)-\frac{s}{\delta}Sin\,s\frac{\pi}{2}Sin\left(s\frac{\pi}{2}+\frac{\pi}{4}\right)}+O(s^{-2}e^{|t|(y-x)}) & 0\le y<\frac{\pi}{2}\le x\le\pi \\ \dfrac{Sin\,s(x-\pi)\left[\frac{-1}{\delta}Sin\,s\left(y-\frac{\pi}{2}\right)+\delta Cos\,s\left(y-\frac{\pi}{2}\right)\right]}{\delta s\,Cos\,s\frac{\pi}{2}+\frac{s}{\delta}Sin\,s\frac{\pi}{2}}+O(s^{-2}e^{|t|(y-x)}) & \frac{\pi}{2}<y\le x\le\pi \\ \dfrac{Cos\left(sx+\frac{\pi}{4}\right)\left(Sin\,s\left(y-\frac{\pi}{2}\right)\right)}{s\,Cos\left(s\frac{\pi}{2}+\frac{\pi}{4}\right)}+O(s^{-2}e^{|t|(y-x)}) & 0\le x\le y<\frac{\pi}{2} \\ \dfrac{Sin\,s(y-\pi)Cos\left(sx+\frac{\pi}{4}\right)}{\delta s\,Cos\,s\frac{\pi}{2}Cos\left(s\frac{\pi}{2}+\frac{\pi}{4}\right)-\frac{s}{\delta}Sin\,s\frac{\pi}{2}Sin\left(s\frac{\pi}{2}+\frac{\pi}{4}\right)}+O(s^{-2}e^{|t|(y-x)}) & 0\le x<\frac{\pi}{2}<y\le\pi \\ \dfrac{Sin\,s(y-\pi)\left[\frac{-1}{\delta}Sin\,s\left(x-\frac{\pi}{2}\right)+\delta Cos\,s\left(x-\frac{\pi}{2}\right)\right]}{\delta s\,Cos\,s\frac{\pi}{2}+\frac{s}{\delta}Sin\,s\frac{\pi}{2}}+O(s^{-2}e^{|t|(y-x)}) & \frac{\pi}{2}<x<y\le\pi \end{cases}$$

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